

# Gases

You can do everything with the Perfect Gas Law  
(The Equation of State)...

$$pV = mRT$$

Where

**p** = pressure (Pa) **absolute!**

**V** = volume (m<sup>3</sup>)

**m** = mass of gas (kg)

**R** = 8.314472 / M (J/kgK) where M = relative molecular mass (no units)

**T** = temperature (K) **Kelvin!**

## Example:

Find R for CO<sub>2</sub> (molecule)

$$M = 12 + 2 \times 16 = 44 \text{ g/mole}$$

A mole is a big number (Avagadro's number =  $6.0221413 \times 10^{23}$ )

$$R = 8.314472 / M$$

$$= 8.314472 / 44$$

$$= 0.188965 \text{ J/kgK}$$

Look on chart for CO<sub>2</sub>: R = 0.189

## The General Gas Equation

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

This is a quicker way when dealing with 2 *states* using the same amount of gas. (m and R stay the same).

This comes directly from the Equation of State...

$$p_1 V_1 / \cancel{mR} T_1 = p_2 V_2 / \cancel{mR} T_2$$

$$p_1 V_1 / T_1 = p_2 V_2 / T_2$$

Should we use  $pV = mRT$  or  $p_1V_1 / T_1 = p_2V_2 / T_2$  ???

When to use the Equation of State...

$pV = mRT$ :

- Anything about mass **m** or gas type **R**
- Only 1 state

When to use the General Gas Equation...

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

- There are 2 states
- Not given mass **m** or gas type **R**

# 1st law of thermo + gases

Thursday, 10 November 2011  
11:29 AM

Process	Relationship between $p$ , $V$ , $T$	Work ( $W$ )	Internal-energy change ( $U_2 - U_1$ )	Heat ( $Q$ )
constant pressure	$p = \text{constant}$ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$	$p(V_2 - V_1)$	$mc_v(T_2 - T_1)$	$mc_p(T_2 - T_1)$

Constant Pressure...

$$\Delta U = Q - W$$

$$\Delta U = mc_v(T_2 - T_1) \quad (\text{true for everything... Isothermal, Isobaric etc...})$$

$$W = p\Delta V$$

$$Q = \Delta U + W$$

$$= mc_v(T_2 - T_1) + p\Delta V$$

$$= mc_p(T_2 - T_1) \quad (\text{so } c_p \text{ takes work into account}).$$

Gas	Formula	$c_p$ Constant Pressure Specific Heat Capacity (kJ/kgK) at 20°C	$c_v$ Constant Volume Specific Heat Capacity (kJ/kgK) at 20°C	$\gamma$ Ratio of Specific Heats $\gamma = c_p / c_v$	R Characteristic Gas Constant (kJ/kgK) $R = c_p - c_v$
Acetone		1.47	1.32	1.11	0.15
Acetylene	$C_2H_2$	1.69	1.37	1.232	0.319
Air		<b>1.005</b>	<b>0.718</b>	<b>1.40</b>	<b>0.287</b>
Ammonia	$NH_3$	2.19	1.66	1.31	0.53
Argon	$Ar$	0.520	0.312	1.667	0.208
Benzene	$C_6H_6$	1.09	0.99	1.12	0.1

That is why  $c_p$  is larger than  $c_v$

## Example: Gas Eqn

Tuesday, 1 November 2011  
6:37 PM

Q1: A certain amount of gas fills 4.9 m<sup>3</sup> at 128°C and 28 kPa (gauge). What will be its volume at standard temp (20°C) and pressure (101.3 kPa)?

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Conversions;

$$T_2 = 20 + 273 = 293 \text{ K}$$

$$p_1 = 28000 + 101300 = 129300 \text{ Pa}$$

$$T_1 = 128 + 273 = 401 \text{ K}$$

$$p_2 = 101300 \text{ Pa}$$

$$p_1 V_1 / T_1 = p_2 V_2 / T_2$$

$$V_2 = T_2 p_1 V_1 / (T_1 p_2)$$

$$= 293 * 129300 * 4.9 / (401 * 101300) = 4.5699 \text{ m}^3$$

Q2: Compressed air ( $R=287 \text{ J/kgK}$ ) fills a tank of diam 398 mm x length 2.5 m. Pressure is 1.22 MPa (gauge), temperature is 25°C. Calculate air mass.

Where

$p$  = pressure (Pa) **absolute!**

$V$  = volume (m<sup>3</sup>)

$m$  = mass of gas (kg)

$R$  = 8.314472 /  $M$  (J/kgK)

$T$  = temperature (K) **Kelvin!**

$$pV = mRT$$

$$m = pV/RT$$

Convert:

$$V = \pi * (0.5 * 0.398)^2 * 2.5 = 0.311026 \text{ m}^3$$

$$p = 1.22\text{E}6 + 101.3\text{E}3 = 1321300 \text{ Pa}$$

$$T = 273 + 25 = 298 \text{ K}$$

$$R = 0.287 * 1000 = 287 \text{ J/kgK} \text{ (If you looked up air in the table)}$$

$$m = pV/RT$$

$$= 1321300 * 0.311026 / (287 * 298) = 4.805073 \text{ kg}$$

# The 7 part question!

Tuesday, 1 November 2011  
7:07 PM

Q7: Initial temperature = 16°C, initial height 878 mm.  
Cylinder diameter = 610 mm. Piston mass = 17.9 kg. Find mass of air.

$$pV = mRT$$

$$m = pV/RT$$

Convert:

$$\text{Area} = \pi \cdot (0.5 \cdot 0.610)^2 = 0.292247 \text{ m}^2$$

$$V = 0.292247 \cdot 0.878 = 0.256593 \text{ m}^3$$

$$p = F/A = (17.9 \cdot 9.81) / (0.292247) = 600.858178 \text{ Pa (Gauge!!!)}$$

$$= 600.858178 + 101.3 \text{E}3 = 101900.86 \text{ Pa (Absolute)}$$

$$T = 273 + 16 = 289 \text{ K}$$

$$R = 0.287 \cdot 1000 = 287 \text{ J/kgK}$$

$$m = pV/RT$$

$$= 101900.86 \cdot 0.256593 / (287 \cdot 289) = 0.315241 \text{ kg}$$

Where

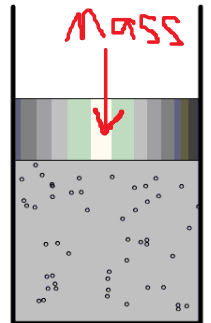
p = pressure (Pa) **absolute!**

V = volume (m<sup>3</sup>)

m = mass of gas (kg)

R = 8.314472 / M (J/kgK)

T = temperature (K) **Kelvin!**



Q8: (cont) Initial temperature = 16°C, initial height 878 mm. Cylinder diameter = 610 mm. Piston mass = 17.9 kg. Expansion = 355 mm. Determine final temperature (°C).

Method 1: Using **pV = mRT**

Some properties same as previous question;

$$p = 101900.86 \text{ Pa (Absolute)}$$

$$m = 0.315241 \text{ kg}$$

$$V_2 = (878+355)/878 \cdot V_1 = (878+355)/878 \cdot V_1$$

$$= 1.4043 \cdot V_1 = 1.4043 \cdot 0.256593 = 0.360334 \text{ m}^3$$

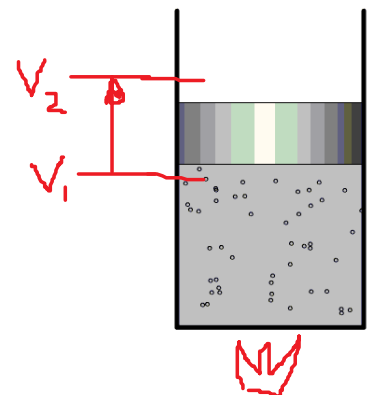
From **pV = mRT...**

$$T = pV/mR$$

$$= 101900.86 \cdot 0.360334 / (0.315241 \cdot 287)$$

$$= 405.84341 \text{ K}$$

$$= 405.84341 - 273 = 132.84 \text{ °C}$$



Method 2: Using

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Some properties same as previous question;

$p = 101900.86 \text{ Pa (Absolute)}$

$m = 0.315241 \text{ kg}$

$$V_2 = (878+355)/878 V_1 = (878+355)/878 V_1 \\ = 1.4043 * V_1 = 1.4043 * 0.256593 = 0.360334 \text{ m}^3$$

$$p_1 V_1 / T_1 = p_2 V_2 / T_2$$

$$T_2 / p_2 V_2 = T_1 / p_1 V_1$$

$$T_2 = T_1 p_2 V_2 / p_1 V_1 \quad (\text{pressures are the same...})$$

$$= T_1 * 1.4043$$

$$= 289 * 1.4043 = 405.8427 \text{ K}$$

$$= 405.8427 - 273 = 132.8427 \text{ }^\circ\text{C} \quad \checkmark \text{ Same!}$$

Q9: (cont) Initial temperature =  $16^\circ\text{C}$ , initial height 878 mm. Cylinder diameter = 610 mm. Piston mass = 17.9 kg. Expansion = 355 mm. Determine heat flow. (+in, -out)

We need Q

From 1st law of Thermo:

$$\Delta U = Q - W$$

But when the substance is constant mass and state

$$\text{Non-flow, No phase change, Constant } c; \quad U_2 - U_1 = mc\Delta T$$

Where

$U$  = internal energy (J)

$m$  = mass (kg)

$c$  = specific heat capacity (J/kgK)

$$\text{So... } \Delta U = mc\Delta T$$

$$mc\Delta T = Q - W$$

So in order to find the heat Q, we must calculate the work  $W = p\Delta V$  OR...

Ignore the work and use  $c_p$  Constant Pressure Specific Heat Capacity

Gas	Formula	$c_p$ Constant Pressure Specific Heat Capacity (kJ/kgK)	$c_v$ Constant Volume Specific Heat Capacity (kJ/kgK)
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Gas	Formula	$c_p$ Constant Pressure Specific Heat Capacity (kJ/kgK) at 20°C	$c_v$ Constant Volume Specific Heat Capacity (kJ/kgK) at 20°C	
Acetone		1.47	1.32	1
Acetylene	$C_2H_2$	1.69	1.37	1
Air		1.005	0.718	1

(This saves a lot of time for standard engineering problems).

$$Q = mc_p \Delta T$$

$$= 0.315241 * 1005 * (132.8427 - 16) = 37017.78 \text{ J}$$

(This is the amount of heat that was put into the gas in order to make it expand)

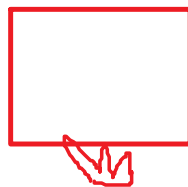
Q10: (cont) Initial temperature = 16°C, initial height 878 mm. Cylinder diameter = 610 mm. Piston mass = 17.9 kg. Expansion = 355 mm. Find internal energy change.

$$mc\Delta T = Q - W$$

Check if this works....

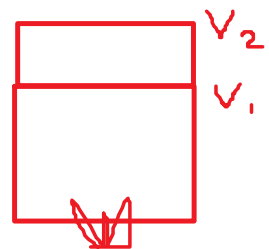
$$\text{LHS} = mc_v \Delta T$$

$$= 0.315241 * 718 * (132.8427 - 16) = 26446.53 \text{ J}$$



Constant Volume Heating  
(No volume change = no work)

$$\Delta U = Q$$



Constant Pressure Heating  
(Volume changes = work)

$$\Delta U = Q - W$$

$$\text{RHS} = Q - W$$

$$Q = 37017.78 \text{ J}$$

$W = p\Delta V$  This is the work AGAINST the atmosphere...

$$= 101300 * (0.360334 - 0.256593)$$

$$= 10508.9633 \text{ J}$$

PLUS... We lifted up the piston...

$$W = Fd = 17.9 * 9.81 * 0.355 = 62.3376 \text{ J}$$

$$W_{\text{tot}} = 10508.9633 + 62.3376 = 10571.3 \text{ J}$$

$$Q - W = 37017.78 - 10571.3 = 26446.48 \text{ J}$$

Q11: (cont) Initial temperature = 16°C, initial height 878 mm. Cylinder diameter = 610 mm. Piston mass = 17.9 kg. Expansion = 355 mm. Find work of the system. (-in, +out)

Easy... (Did this already when we checked the previous question)

$W = p\Delta V$  This is the work AGAINST the atmosphere...

$$= 101300 \cdot (0.360334 - 0.256593)$$

$$= 10508.9633 \text{ J (against atmosphere only)}$$

PLUS... We lifted up the piston...

$$W = Fd = 17.9 \cdot 9.81 \cdot 0.355 = 62.3376 \text{ J}$$

$$W_{\text{tot}} = 10508.9633 + 62.3376 = 10571.3 \text{ J}$$

OR... From 1st Law...

$$\Delta U = Q - W$$

$$W = Q - \Delta U$$

$$= Q - mc_v \Delta T$$

$$= 37017.78 - 26446.53$$

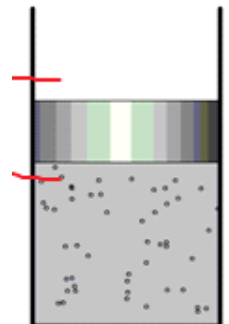
$$= 10571.25 \text{ J}$$

Q12: (cont) Initial temperature = 16°C, initial height 878 mm. Cylinder diameter = 610 mm. Piston mass = 17.9 kg. Expansion = 355 mm. Find work of piston. (-in, +out)

Easy - did this before...

PLUS... We lifted up the piston...

$$W = Fd = 17.9 \cdot 9.81 \cdot 0.355 = 62.3376 \text{ J}$$



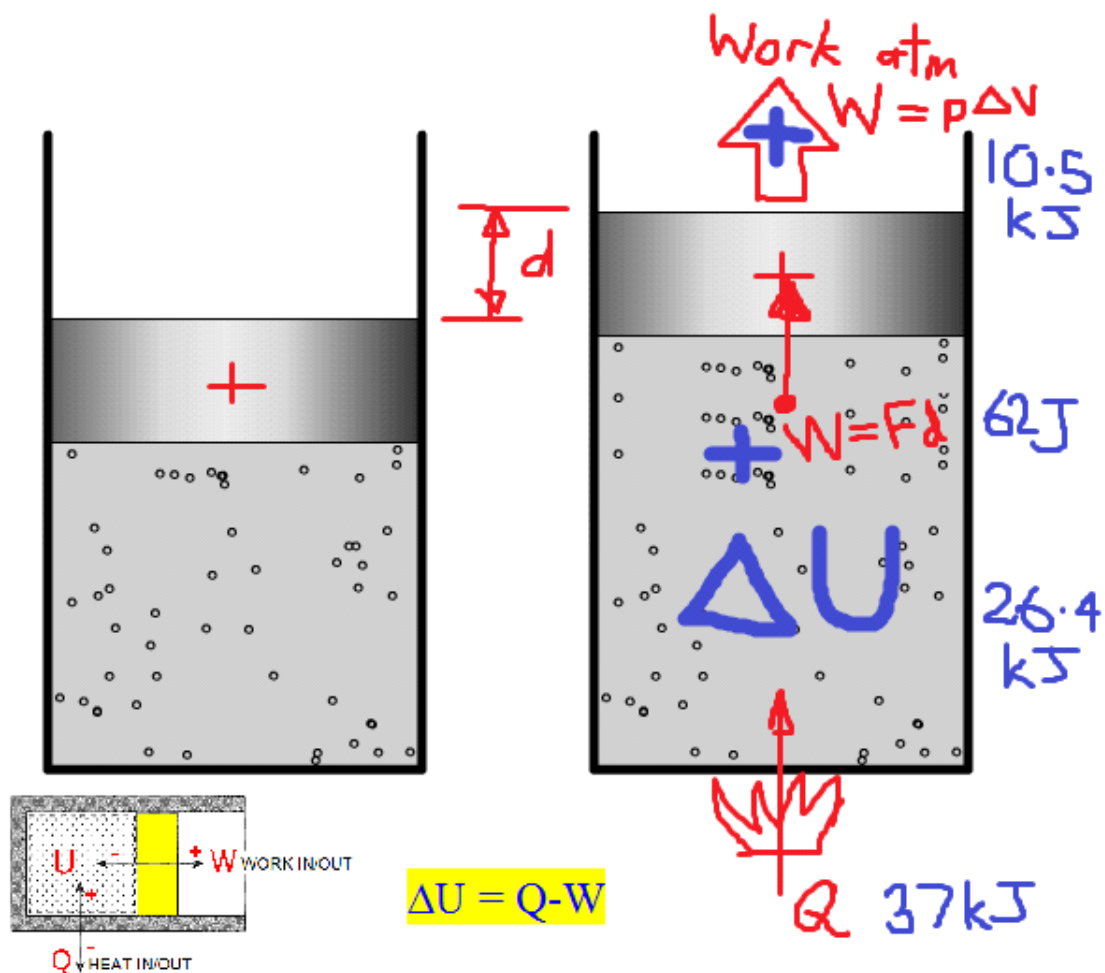
Q13: (cont) Initial temperature = 16°C, initial height 878 mm. Cylinder diameter = 610 mm. Piston mass = 17.9 kg. Expansion = 355 mm. Find work of atmosphere. (-in, +out)

Easy... repeat from above....

$W = p\Delta V$  This is the work AGAINST the atmosphere...

$$= 101300 \cdot (0.360334 - 0.256593)$$

$$= 10508.9633 \text{ J (against atmosphere only)}$$



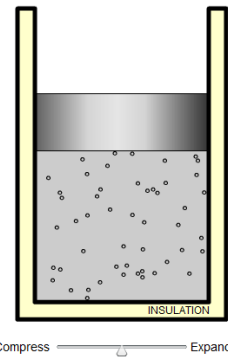
The internal energy INCREASES if heat goes in, and if work goes IN.

In our case, heat goes IN and work goes OUT, but more heat than work, so internal energy INCREASED.

# Polytropical!

Tuesday, 8 November 2011  
6:45 PM

Well Adiabatic actually.



Q20: Air at 28 kPa and 24°C fills an insulated cylinder of 3.74 litres. It is compressed down to 0.53 litres. (a) What is the final temperature? (°C)

~~$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$~~

This does not work!!!! (No heat flow = ADIABATIC)

The pressure and volume affect each other by the equation:

$pV^n = \text{constant}$

Go back to the table...

	$p, V, T$	$W$	$\Delta U$	$Q$
polytropic	$pV^n = \text{constant}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ $\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$	$\frac{p_1 V_1 - p_2 V_2}{n - 1}$	$mc_v(T_2 - T_1)$	$Q = W + U_2 - U_1$
adiabatic	as for polytropic with $n = \gamma$	as for polytropic with $n = \gamma$	$mc_v(T_2 - T_1)$	0

$\text{Air} = \gamma = 1.4 = n$

Adiabatic is a type of Polytropic, where  $n = \gamma$  ( $n = \text{polytropic index} = 1.4 = \text{AIR}$ )

Q20: Air at 28 kPa and 24°C fills an insulated cylinder of 3.74 litres. It is compressed down to 0.53 litres. (a) What is the final temperature? (°C)

$$T_2 = T_1 \cdot (V_1/V_2)^{(n-1)}$$

Convert:

$$T_1 = 273.15 + 24 = 297.15 \text{ K}$$

$$V_1 = 3.74/1000 = 0.00374 \text{ m}^3$$

$$V_2 = 0.53/1000 = 0.00053 \text{ m}^3$$

$$T_2 = 297.15 \cdot (0.00374/0.00053)^{(1.4-1)}$$

$$= 649.25309 \text{ K}$$

$$= 649.25309 - 273.15 = 376.10309 \text{ °C}$$

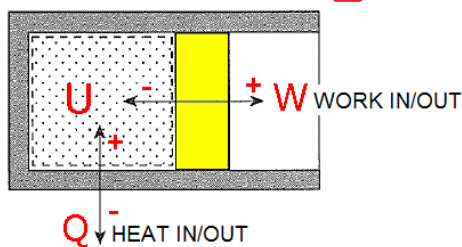
Q21: (cont) Air at 28 kPa and 24°C fills an insulated cylinder of 3.74 litres. It is compressed down to 0.53 litres. (b) Determine the final pressure.

$$p_2 = p_1 \cdot (T_2/T_1)^{n/(n-1)}$$

$$\begin{aligned} \text{Convert: } P_1 &= 28000 + 101300 = 129300 \text{ Pa} \\ &= 129300 \cdot (649.25309/297.15)^{(1.4/(1.4-1))} \\ &= 1.99357 \text{ MPa} \end{aligned}$$

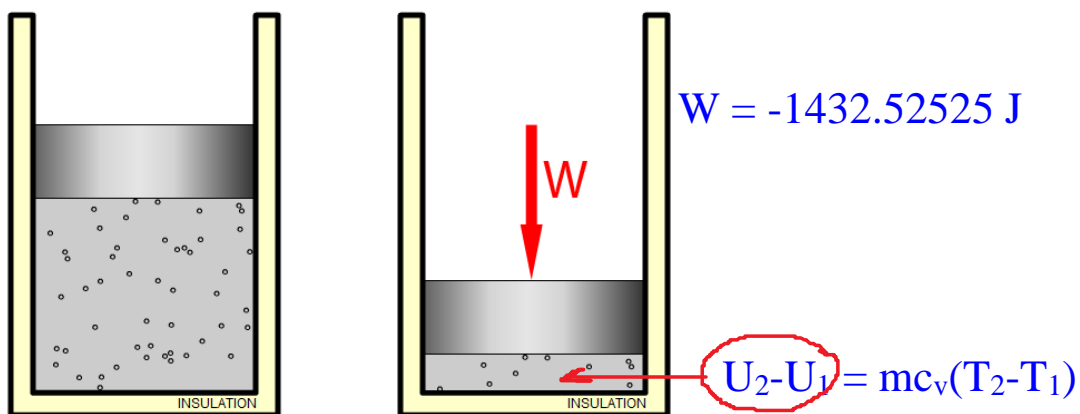
Q22: (cont) Air at 28 kPa and 24°C fills an insulated cylinder of 3.74 litres. It is compressed down to 0.53 litres.  
(c) How much work was done BY the gas?  
(Hint: Watch  $\pm$  signs)

$$\begin{aligned} W &= (p_1 V_1 - p_2 V_2)/(n-1) \\ &= (129300 \cdot 0.00374 - 1.99357 \text{E}6 \cdot 0.00053)/(1.4-1) \\ &= -1432.52525 \text{ J} \end{aligned}$$



So the work is NEGATIVE????? YES!!!!!!!!!!

Work done BY the gas is backwards (negative)...



CHECK...

Find change of internal energy for gas;

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

Find m:

$$pV = mRT$$

Where

**p** = pressure (Pa) **absolute!**

**V** = volume (m<sup>3</sup>)

**m** = mass of gas (kg)

**R** = 8.314472 / M (J/kgK) where M = relative molecular mass (no units)

**T** = temperature (K) **Kelvin!**

Find mass at state 1:

$$m = pV/RT = (129300 \times 0.00374) / (287 \times 297.15) = 0.00567 \text{ kg}$$

Back to 1st law of thermo:

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

$$= 0.00567 \times 718 \times (649.253 - 297.15)$$

$$= 1433.43 \text{ J}$$

**Adiabatic** = insulated = no heat flow:

$$U_2 - U_1 = Q - W$$